# THAI NGUYEN UNIVERSITY UNIVERSITY OF EDUCATION

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# TEACHING MATHEMATICAL LOGIC IN THE ORIENTATION OF CONTRIBUTING TO DEVELOPING MATHEMATICAL LANGUAGE COMPETENCE FOR STUDENTS OF MATHEMATICS EDUCATION

Major: Theory and Methods of Teaching Mathematics Code: 9140111

# **DISSERTATION SUMMARY**

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# The dissertation can be found at:

- National Library of Vietnam.
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# THE AUTHOR'S PUBLICATIONS RELATED TO THE DISSERTATION TOPIC

- Nguyen Thi Chung (2016), Some difficulties and mistakes of students when solving problems related to geometrical representations of complex numbers, Journal of Education, Ministry of Education and Training, pp.29-31. No. 374.
- Nguyen Thi Chung (2017), Investigate the real situation of mathematical language competence of students of mathematics education at universities. Journal of Education, Ministry of Education and Training, pp.51-63, No. 405.
- 3. Bui Thi Hanh Lam, Nguyen Thi Chung (2018), *The conceptions of students of mathematics education on the components of mathematical language competence at universities*, Journal of Education, Ministry of Education and Training , p. 427.
- Nguyen Thi Chung (2018), Some measures to contribute to developing mathematical language competence for students of mathematics education through teaching Mathematical Logic, Journal of Educational Management, page 11, No. 2.
- 5. Nguyen Thi Chung, Do Thi Hoai, Dao Hong Dieu, Nguyen Thi Ngoc Hang (2020), Experiment on the measures to develop mathematical language competence for students of mathematics education through teaching Mathematical Logic, Journal of Educational Management, page 12, No.4.

### INTRODUCTION

#### 1. Reason for choosing the research topic

Pedagogical universities are institutions which train teachers, meeting the requirements of society. The competence of students and teachers reflects the quality of training of pedagogical universities. The current reform of general education has placed great demands on pedagogical universities in renovating content, programs, methods, organizational forms of teaching, testing and evaluation to help students acquire basic knowledge and skills to meet the requirements of education in the new period.

According to Circular 32/2108/TT-BGDĐT, Mathematics Education focuses on developing learners' competencies. In particular, the core mathematical competencies that need to be developed for students are the competence of mathematical thinking and reasoning, the competence of mathematical modeling, the competence of solving mathematical problems, the competence of mathematical communication, and the competence to use tools and means. In order to develop competencies for learners, the professional competence of teachers also needs to be improved. Therefore, the development of professional competence for SME should also be stressed in the training process at pedagogical universities.

The reality of teaching at universities shows that the competence of using mathematical language (ML) of students of maths education (SME) is limited and not paid enough attention to; students do not have a clear sense of the importance of the competence of using ML. Many SME do not really understand the meaning of ML; they use ML incorrectly, arbitrarily in learning, teaching and researching maths. University lecturers have focused on developing students' ML competence in their teaching, research and vocational training, however, it is not uniform in all modules and empirical; there are no specific ways to guide lecturers, especially those in basic Maths subjects.

Teaching Mathematical logic in the training of Math teachers not only helps students understand mathematical subjects and relations, but also helps students develop logical thinking in math learning and research, address the problem in a concise and accurate way, know how to apply mathematical logic to solve related problems in mathematics, in practice, in mathematics teaching and research.

For the above reasons, we selected the research topic: "*Teaching Mathematical Logic in the orientation of contributing to developing mathematical language competence for students of mathematics education*".

### 2. Research Aims

Identify the components of the competence of using mathematical language among students of maths education. Propose pedagogical measures in teaching Mathematical logic in a way that contributes to developing the competence of using ML for students of maths education.

# 3. Research subjects and objects

# 3.1. Research subjects

Elements of ML competence of SME, measures in teaching Mathematical logic towards contributing to the development of ML competence for SME.

# 3.2. Research objects

The teaching process at universities with pedagogical departments in the orientation of developing ML competence for SME.

**4. Research questions:** Mathematical language? The competence of using mathematical language? The competence of using mathematical language of students of maths education? How to develop the competence of using mathematical language for students of maths education?

# 5. Scientific hypothesis

Based on theoretical and practical basis, some components of ML competence of SME can be identified. On that basis, if appropriate pedagogical measures in teaching Mathematical Logic can be proposed and implemented, they will contribute to developing the competence of using ML for SME, meeting the requirements of renovating mathematics teaching at high schools in the competence-based approach.

# 6. Research tasks

6.1. Learn about language, ML, teachers' professional competence, and ML competence.

6.2. Identify the components of ML competence of SME; ML competence levels of SME.

6.3. Investigate the relationship between teaching Mathematical logic with the development of ML competence of SME.

6.4. Investigate the reality of teaching Mathematical Logic and the situation of ML competence of SME in universities.

6.5. Propose pedagogical measures to contribute to developing the competence of using ML for SME.

6.6. Conduct pedagogical experiment to clarify the feasibility and effectiveness of the proposed measures in the thesis.

# 7. Research scope

The research is done in the field of teaching Mathematical Logic in the direction of developing the competence of using ML for SME at the University of Pedagogy (hereinafter referred to as students of maths education).

# 8. Research methods

8.1. Methods of theoretical research: Study documents and works related to the research problem.

8.2. Survey method: Design and use questionnaires, conduct interviews to understand the reality of teaching Mathematical logic at university and the situation of the competence of using ML of SME.

8.3. Mathematical statistics method: process survey data, diagnostic data (before the experiment) and post-experimental results data.

8.4. Expert method: Consult with experts about issues within the scope of the thesis research.

7.5. Pedagogical experiment: conduct experiments to test the feasibility and effectiveness of pedagogical measures proposed in the thesis.

# 9. The points to be defended

9.1. The perception of ML competence of SME, the components of ML competence of SME.

9.2. The measures proposed in teaching Mathematical logic which contributes to developing the competence of using ML for SME are feasible and effective.

# 10. The contributions of the thesis

10.1. Theoretical contributions

The thesis has:

- developed the concept of the competence of using ML of SME. On the basis of analyzing teaching and learning activities of students, the dissertation has also identified elements of ML competence of SME.

- analyzed the relationship between teaching Mathematical logic and the development of ML competence of SME.

- proposed some pedagogical measures in teaching Mathematical logic towards contributing to developing the competence of using ML for SME.

10.2. Practical contributions

- The system of pedagogical measures can help students to be more conscious and perform effectively in the process of teaching Mathematical Logic, helping students to better exploit the knowledge of Mathematical Logic in the process of teaching Mathematics.

- The system of examples, exercises, topics in the thesis is a good material for lecturers and students to refer to and apply in training SME in the direction of contributing to developing the competence of using ML.

# **11. Structure of the thesis**

In addition to the introduction, conclusion, references and appendices, the thesis consists of three chapters:

Chapter 1: Theoretical and practical basis

**Chapter 2:** Some pedagogical measures that help SME develop their ML competence through teaching Mathematical Logic.

Chapter 3: Pedagogical experiment

# **Chapter 1. THEORETICAL AND PRACTICAL BASIS**

# **1.1.** Overview of research on mathematical language and the competence of using mathematical language

# 1.1.1. Overview of research on mathematical language In the world:

According to A.A.Stôliar "Using modern ML (Mathematical Logic) in teaching mathematics in high schools today is an issue that is widely debated. In order to solve it effectively in terms of pedagogy, it is necessary to have long-term experiments, and even the teachers must properly grasp this language".

Author Martin Hughes (1986) studied the use of arithmetic symbols in students' math learning and the difficulties of students when learning this ML.

Pimm (1987), Laborde (1990), Ervynck (1982), studied ML in students' mathematical learning. These researchers have affirmed that without ML, there will be no communication process in math class and mathematics cannot take place. They realized that the language of mathematics was really a difficulty and obstacle in learning math because the language of mathematics is different from the language of everyday use.

Eula Ewing Monroe and Rebent Panchyshyn (1995) studied the issue of vocabulary, symbols of ML and raised the need of using ML in developing mathematical concepts and theorems.

Birgit Pepin (2007) studied the UK's national curriculum for ML including the correct use of ML in mathematics and in practice.

# In Vietnam:

The author Ha Si Ho (1990) has argued that ML is primarily a sign-language, ML is not a "spoken" language but rather a "written" language.

Hoang Chung (1994) studied the use of ML in mathematics textbooks at secondary school level.

According to the authors Pham Van Hoan, Nguyen Gia Coc and Tran Thuc Trinh (1981), ML is different from the natural language in that it is compact, capable of accurately expressing mathematical ideas, very suitable for expressing general rules since ML uses variable languages.

According to the author Phan Anh, "ML mainly involves using signs", so the development of ML is closely associated with the development of mathematical signs.

Thus, since the 1970s the ML has been systematically studied in a close relationship with the natural language. Researchers have confirmed that without

*ML*, there will be no communication process in math class, which confirms the important role of *ML* in teaching and learning mathematics.

# 1.1.2. Overview of research on the competence of using mathematical language In the world:

According to Ken Winogand and Karen M. Higgins (1994), it is possible to support the development of ML for students by providing them with a system of tools such as numbers, algebraic symbols, graphs, charts, models, equations, signs, images ...

David Chard (2003) also studied the vocabulary of ML, formulated a vocabulary development plan in learning mathematics and found that ML is an important means to help students develop new concepts.

Mihaela Singer (2007), who studied ML in Romanian mathematics education program, made the following statement: Communication in ML is one of the four goals of math education implemented from the first grade to the last grade of general education; ML is a means to express mathematical knowledge.

Charlene Leader House (2007) studied the students' ML, their understanding and use of ML.

Glenda Anthony and Margaret Walsaw studied innovation in teaching math in schools

According to N.G. Trennuwsepxiki, "Practicing the correct language skills is training the correct thinking. When students work on an exercise, paying attention to each question, word, dot, comma, they are thinking. Among exercises for students, it is advisable to design exercises that require formulas to be translated into ordinary languages to avoid formalism and to practice using language correctly."

According to Rheta N. Rubenstein (2009), mathematical communication is an important content in the goal of mathematical education.

# In Vietnam:

According to Nguyen Van Thuan, in order to develop ML competence for students, it is necessary to train them how to use mathematical terms and symbols to express mathematical propositions.

Nguyen Ba Kim (2011) argued that developing logical thinking and exact language for students through maths can be done in three closely related directions.

According to Nguyen Huu Tinh (2008), ML is flexible, a mathematical symbol in different contexts can express different contents.

At the national workshop on general education, Tran Luan (2011), when referring to students' mathematical competencies, claimed that elements of ML were considered in describing mathematical competencies of students.

According to Nguyen Huu Hau (2011), to develop ML for students in the process of teaching mathematics at high schools, teachers need to pay attention to

training students to understand correctly, use correctly and reasonably the language of set theory and mathematical logic.

Phan Anh (2011) said that the ability to use natural language and ML is the prerequisite for other components of mathematical competency in practical situations of high school students.

Tran Ngoc Bich (2013) studied some measures to help students in the first grade of elementary school to effectively use ML.

Le Van Hong helped SME develop their ML competence in teaching mathematics at high school directly through the exploitation of coursebooks and teaching materials on methodology of teaching maths.

According to Vu Thi Binh (2015), in training SME, there are many opportunities to exploit and develop mathematical representations, which can help students use and create mathematical representations, guiding them to formulate and develop competence.

The above studies have confirmed the role of ML competence in developing mathematical competence for both university students and school students. The above research results indicate that to develop the competence of using ML for school students it is necessary to study the development of ML competence for SME. This is an important issue in the process of teacher training, having scientific significance and practical value. However, up to now, in our country, there has not been any specific research project on contributing to developing competence of using ML for SME through teaching Mathematical Logic, thereby contributing to the development of professional competence for SME in the current situation of educational innovation in high school.

# 1.2. Competence and professional competence of teachers

# 1.2.1. The concept of competence

Based on the different views on the above competencies, we believe that competence is the ability to mobilize and use resources to effectively solve problems that arise in a given situation.

# 1.2.2. Professional competence of teachers

Professional competence of teachers is understood as pedagogical competencies. According to psychologists, pedagogical competence is the reflection of certain personality traits that meet the requirements of teaching and education.

**1.2.3. Regional standards for Maths teachers in Southeast Asia (SEARS - MT):** Analyzing these standards, we found that SEARS - ML focuses on teachers' capacity to develop thinking for students, teachers' knowledge of mathematics and their math teaching skills. These are competencies that need to be developed for teachers.

# 1.2.4. Outcome standards for Bachelors of Mathematics education

1.2.4.1. Outcome standards for bachelors of education: In the framework of outcome standards for bachelors of education, there are many measures to develop competence, particularly in standards 4, 5, 8. One of these measures is

through the development of language competence for students in the teaching process at university.

1.2.4.2. Outcome standards for the math teacher training program

Studying the outcome standards for the math teacher training program at some universities, it is possible to realize the following common points: ML competence is one of the necessary competences for SME so that after graduation, students will be able to perform the role of math teacher in high school in the current situation and in the future.

# 1.3. Mathematical language

# 1.3.1. The concept of language

Based on the concepts of language, we think that language is a system of symbols, words, and rules that combine them to make a common communication tool for the community in life and in study.

# 1.3.2. The concept of mathematical language

In this thesis we conceive that: *ML in teaching mathematics at high school is the language of mathematical science, including mathematical terms, signs and symbols (such as figures, diagrams, graphs, ...) and the rules that combine them to express mathematical matters and relationships while speaking, writing or thinking.* 

# 1.3.3. Functions of mathematical language

ML functions as the means of communication and the tool of thinking.

Therefore, communication is an important function in learning, teaching and researching mathematics. In Maths classroom at the university, there is an exchange of information between lecturers and students, between individual students and the class, between individual students and individual students for the purpose of helping students understand mathematical concepts and theorems. This helps students develop ML competence.

# 1.3.4. Characteristics of mathematical language

According to Pham Van Hoan and other authors, ML has important characteristics: brevity; ability to accurately express mathematical thoughts; ability to generalize general rules.

Thus, it can be understood that ML include terms, symbols, figures, diagrams, charts, graphs, semantics and syntax. Therefore, ML facilitates the process of exchanging, arguing, reasoning, explaining and communicating ideas in Mathematics and in thinking.

# **1.4.** Mathematical language competence of students of mathematical education

# 1.4.1. Mathematical language competence

Based on the concepts of competence and language use, we believe that: the competence of using language is the ability to acquire and process information related to languages; the ability to manipulate language in learning, in communication, in teaching and research. On the basis of research on the capacity to use language and on ML, we think that: *ML competence is the ability to receive and process information related to ML; the ability to apply ML in learning, in mathematical communication and in mathematical representation, in mathematical research; the ability to use ML flexibly in real life.* 

# 1.4.2. Mathematical language competence of students of mathematical education

1.4.2.1. Characteristics of ML and ML competence of SME

Through studying theoretical foundations, surveying the situation and practice of teaching Maths to SME, we find that the characteristics of SME' ML competence are: ML competence in the process of learning, researching and teaching Maths; ability to detect, predict and correct logical errors in solving problems; ability to assess the use of ML of their own and of students. These are also necessary competencies of Math teachers in high school.

1.4.2.2. Identify the components of ML competency of SME

In order to identify the components of ML competency of SME, we based on some foundations:

• Foundation 1: Regulations on professional standards of teachers of general education institutions [Circular No. 20/2018/TT-BGDDT]; Regional standards for Maths teachers in Southeast Asia (SEARS - MT).

• Foundation 2: Outcome standards for pedagogical university graduates in Vietnam; Outcome standards for teacher training program in Hanoi University of Education, Thai Nguyen University of Education, and Hai Phong University.

• Foundation 3: General Mathematics Education Program.

• Foundation 4: The views of some authors on ML competence.

We believe that the components of ML competence of SME include:

*Component 1: The ability to receive knowledge, understand and correctly use mathematical terms and symbols in learning, teaching and researching Mathematics.* 

Component 2: The ability to use correct mathematical representations in terms of semantics and syntax to solve maths problems, teach math problems and do research on maths.

Component 3: The ability to reason closely and correctly use semantic and syntactic aspects of mathematical reasoning in learning, teaching and researching Maths.

Component 4: The ability to guide and support high school students to use the correct vocabulary, terms, mathematical symbols, mathematical representations and to foster logical thinking for students in the process of teaching Math.

Component 5: The ability to evaluate the use of ML of their own and of students in the process of teaching Math.

*1.4.2.3. Criteria and indicators of the components of ML competence of SME* **Table 1.1: Criteria and indicators of the components of ML competence of SME** 

Criteria	Indicators
The ability to receive knowledge, understand and correctly use mathematical terms and symbols in learning, teaching and researching Mathematics.	<ul> <li>1.1. The ability to listen and understand the content of lectures, arguments, requirements of lectures, and the contents of classmates' presentations when discussing or reporting on topics or projects.</li> <li>1.2. The ability to take notes of lectures and information according to their own understanding, to represent knowledge in their own way of understanding (have a distinct and creative way of taking notes by using mathematical terms, symbols and representations) when learning math.</li> <li>1.3. The ability to use mathematical language (terms, symbols, mathematical representations, etc.) when speaking to argue, explain and present mathematical problems when being asked, when discussing, when reporting seminars, projects or when teaching.</li> </ul>
The ability to use correct mathematical representations in terms of semantics and syntax to solve maths problems, teach math problems and do research on maths.	<ul> <li>2.1. The ability to visualize and diagram relationships of mathematical matters in specific situations.</li> <li>2.2. The ability to use mathematical terms and symbols to express mathematical matters accurately, visually and creatively.</li> <li>2.3. The ability to use mathematical representations properly in terms of semantics and syntax to find for themselves or guide students to find ways of solving mathematical and practical situations.</li> </ul>
	3.1. The ability to deduce reasonably, to use ML to express the rules of reasoning in presenting scientific problems and in teaching Math.

Criteria	Indicators
The ability to reason closely and correctly use semantic and syntactic aspects of mathematical reasoning in learning, teaching and researching Maths.	<ul> <li>3.2. The ability to divide cases in problems; to consider possible cases for research problems, to predict mathematical results based on individual and special cases; to generalize to discover general problems in learning and doing research on Maths.</li> <li>3.3. The ability to train students in special reasoning to find solutions to problems; to train students to make generalized reasoning to find general problems and discover the nature of the problem being studied.</li> </ul>
The ability to guide and support high school students to use the correct vocabulary, terms, mathematical symbols, mathematical representations and to foster logical thinking for students in the process of teaching Math.	<ul> <li>4.1. The ability to use instructional language and guide the thinking process for students.</li> <li>4.2. The ability to design situations to develop logical thinking for students.</li> <li>4.3. The ability to analyze and predict reasoning errors in a solution; to argue against scientific problems in Mathematics and Mathematics education.</li> </ul>
The ability to evaluate the use of ML of their own and of students in the process of teaching Math.	<ul> <li>5.1. The ability to identify their own level of ML, limitations in using ML in learning, teaching or studying Mathematics.</li> <li>5.2. The ability to propose solutions to improve their limitations in using ML in learning, teaching or studying Mathematics.</li> <li>5.3. The ability to identify limitations of their students in using ML and propose solutions to overcome them.</li> </ul>

In this study, we focus on indicators that we think we can prepare through teaching Mathematical Logic in pedagogical universities: 1.1, 1.2, 1.3, 2.1, 2.2, 2.3, 3.1, 3.2, 4.1, 4.2, 4.3, 5.1, 5.2.

Some bases for determining levels of developing ML competence of SME: Applying the method of building mathematical knowledge levels according to PISA. PISA assesses students' ability according to 6 levels of mathematical proficiency; Based on teachers'; Southeast Asian Regional Standards for Mathematics Teacher (SEARS –ML); Based on the outcoce standards of SME of some pedagogical universities and pedagogy faculties of universities; Inheriting previous studies, especially based on the results of Vu Thi Binh's research on standards, criteria and levels of using mathematical representations and mathematical communication competence of students.

We found that among the above 15 indicators of ML competence, there are weak, average, fair, and good levels. On that basis, we quantify them into four levels of ML competence development of SME as follows:

**Level 1:** = Weak (W): This is the lowest level of ML competence. At this level, students are often passive: SME are not yet able to use ML in taking notes, presenting and explaining mathematical contents in simple situations when learning maths. They can not to present and express their ideas in ML.

**Level 2:** = Average (A): SME can use ML in taking notes, presenting and explaining mathematical contents in simple situations when learning maths. At this level, students are still confused and make syntactic as well as semantic errors when arguing and explaining a Math content.

**Level 3:** = Fair (F): SME can use correct ML to summarize, explain and argue about mathematical problems when learning maths, teaching math and studying mathematics. They can use the right ML to present mathematical ideas and solutions convincingly and effectively. They can proficiently converse from ML to natural language and vice versa. They can also assess their own level of ML use.

**Level 4:** = Good(G): SME can use accurately, flexibly and creatively ML in mathematical reasoning, in solving mathematical problems when learning math, teaching math and studying maths. They can converse from natural language to ML to accurately represent solutions to practical problems. They can proficiently use ML to delve deep into mathematical problems. They can assess the level of using ML of their own and of students.

# **1.5.** Potential of developing ML competence for SME through teaching Mathematical Logic

# 1.5.1. Some contents in Mathematical Logic that can develop ML competence for SME

It can be affirmed that ML in documents related to mathematical logic manifests itself in the form of mathematical symbols, terms, figures, diagrams, tables suitable for thinking and language development of first-year SME. Initially, attention was paid to the development of ML competence of SME.

# 1.5.2. Opportunities to develop ML competence for SME through teaching Mathematical Logic

Based on the following foundations: The concept of ML competence of SME; Contents and characteristics of the subject Mathematical Logic; Outcome standards for SME; Professional standards for Math teachers, we believe that: *teaching Mathematical Logic in a way that contributes to developing the competence of using ML for SME is a process where lecturers organize Mathematical logic teaching activities step by step to help students reach levels of ML competence.* 

Through teaching practice, we realize that the following contents in Mathematical Logic can exploit teaching in the direction of contributing to the development of ML competence for SME:

a) Terms and symbols in Mathematical Logic can help SME express problems (concepts, theorems, rules) in a concise and accurate manner.

b) Knowledge of sets, mappings, propositions and propositional functions helps SME understand the matters and relationships of Mathematical Logic, which are the basis of modern Mathematics.

c) Knowledge of propositions and logical reasoning helps SME think logically in learning, teaching and studying Maths.

d) Using ML of set theory, propositions and propositional functions can help SME solve a number of practical problems and practice the conversion between natural language and ML.

e) Knowledge of sets, mappings, propositions and reasoning helps SME detect mistakes in the use of ML and logical reasoning of their own and of students.

# **1.6.** The reality of teaching Mathematical Logic at Universities and the development of ML competence for SME

# 1.6.1. Survey purpose

We conducted a survey to investigate and assess the current state of ML competence of SME in universities to identify basic measures in teaching Mathematical logic to develop ML competence for SME.

# 1.6.2. Survey participants

Survey participants: 126 lecturers who are directly teaching Maths at universities. Math teachers from 6 high schools in Hai Phong city, 148 SME from 5 universities.

# 1.6.3. Time and location of the survey

Time: from February 18, 2016 to April 29, 2016.

Location: 5 universities and 6 high schools.

**1.6.4.** Survey contents: Firstly: lecturers teaching Math at university; Secondly: high school Math teachers; Thirdly: SME.

1.6.5. Survey methods: Observation; Interview; Questionnaires; Data processing.

#### 1.6.6. Survey results and analysis

General conclusions about the current situation of using ML in teaching Mathematical logic in the current period:

Firstly, the survey results obtained from questionnaires, interviews with math lecturers, math teachers and first-year SME on issues related to ML competence of SME show that SME who use ML in learning Math, teaching Math, and studying Math have average level of ML competence. Their awareness of the role of Mathematical logic in the development of students' ML competence is limited; their ability to apply this subject's knowledge in expressing, explaining math problems, and preparing for teaching maths in high schools in the future is not high. Some lecturers, when interviewed, also said that students still have difficulty in "spoken mathematical language", so their ability to express math issues in pedagogical training will face many difficulties. It is necessary to take measures to help students participate in activities to improve the ability of presenting and lecturing on Mathematics in self-study sessions and seminars.

Secondly, the survey results gained from questionnaires and interviews with math lecturers about the reality of teaching Mathematical logic in universities reveals that lecturers have few materials to exploit, expand knowledge and the exercises related to the use ML are limited and unsystematic. Lecturers have not developed specific measures to orient the development of ML competence for SME. Teachers of Mathematical Logic have also been interested in developing ML competence for SME in the process of teaching Math modules in general, but there are no effective concrete measures to help students maximize this competence.

Thirdly, the survey results obtained from questionnaires and interviews with math lecturers in universities and high school teachers show that the concept of ML competence of SME and the expected pedagogical measures to be taken in teaching Mathematical logic towards contributing to the development of ML competence receive the consent of most lecturers and teachers (over 90%).

In summary, ML competence of SME needs to be developed in the process of teaching mathematics and training pedagogical skills to meet the university's outcome standards for SME in the context of education innovation and the 4.0 technology revolution.

## **1.7.** Conclusion for chapter 1

In Chapter 1, we have deat with some of the following issues: competence, professional competencies, language, ML, and ML competence of SME. In particular, the thesis has identified the components of ML competence of SME as the basis for the accomplishing the research tasks in chapters 2 and 3. By studying the resources and on the basis of these research results, we have the theoretical and practical basis to propose five pedagogical measures in teaching Mathematical Logic in the direction of contributing to the development ML competence for SME.

# Chapter 2. PEDAGOGICAL MEASURES WHICH CONTRIBUTE TO THE DEVELOPMENT OF ML COMPETENCE FOR SME THROUGH TEACHING MATHEMATICAL LOGIC

# **2.1.** Orientations for developing measures to develop ML competence for SME through teaching Mathematical Logic

2.1.1. Orientation 1: The measures are taken based on the achievements of modern science and the teaching theory at university.

**2.1.2.** Orientation 2: The pedagogical measures need to make an important contribution in helping SME to be active in awareness, self-discovery, self-solving problems while teaching Mathematical Logic for students.

**2.1.3.** Orientation 3: The proposed measures aim to contribute to developing ML competence for SME, thereby raising their awareness of self-study and scientific research.

**2.1.4.** Orientation 4: The measures are proposed on the basis of the components of ML competence of SME; outcome standards for math teacher training programs of some universities; and professional competence of high school Math teachers.

**2.1.5.** Orientation 5: The system of the measures is built on important principles that are ensuring the goals, contents and standards of knowledge and skills of the Mathematical Logic curriculum.

# 2.2. Measures contributing to developing ML competence for SME

# 2.2.1 Measure 1: Design situations to create cognitive opportunities through self-discovery and problem solving, helping SME to acquire and appropriately use ML when teaching Mathematical Logic

# 2.2.1.1. Purpose of the measure

Encourage SME to learn actively through self-discovery, problemsolving, and acquiring the vocabulary, syntax and semantics of Mathematical Logic, contributing to enriching their ML.

Encourage SME to use ML and natural language flexibly, properly; understand and properly use mathematical symbols, figures, diagrams, charts, tables, representations in the process of teaching about concepts, theorems, rules, and methods.

The purpose of this measure is to help SME develop indicators 1.1, 1.2, 1.3, 2.2, 2.3, 3.1, 3.2 of the ML competence.

# 2.2.1.2. Scientific basis of the measure

According to the current direction of innovating teaching methods, teaching methods need to be oriented towards organizing for learners to take part in selfdiscipline, active, proactive and creative activities. Therefore, it is necessary to encourage learners to study with higher needs and sense of responsibility, study the problems more actively, discuss more to express more mathematical problems, and gain more knowledge.

# 2.2.2.3. Instructions for implementing the measure

First: Design situations, create opportunities for SME to perceive actively, discover and solve problems by themselves, use mathematical vocabulary, symbols and representations correctly in terms of semantics and syntax through teaching mathematical concepts of Mathematical Logic.

Second: Train SME to use ML to discover and solve problems by themselves while teaching theorems and rules of mathematical logic.

**Illustration Example:** After studying the concept of image and image creation of a set through a map in chapter 1 on set theory and mapping, the lecturer raises the follwoing issue: students have proved the property  $f(A \cap B) \subset f(A) \cap f(B)$ , with a map  $f: X \to Y$  and A, B are two subsets of the set X. In the case of changing the intersection into the union of two sets, does the above inclusion change?

Students predict the answer:  $f(A \cup B) = f(A) \cup f(B)$ ?

*Step 1:* The lecturer raises the above problem, which is to give students a problematic situation.

*Step 2:* The lecturer organizes activities for students to self-perceive and comprehend formulas.

- Students check the correctness of the inclusion formula  $f(A \cup B) \supset f(A) \cup f(B)$  (1) and  $f(A \cup B) \subset f(A) \cup f(B)$  (2).

- The lecturer asks a student to use mathematical signs for an image of a set, a union of two sets, through a mapping to check the inclusion (1):

$$y \in f(A) \cup f(B) \Rightarrow \begin{bmatrix} y \in f(A) \\ y \in f(B) \end{bmatrix} \Rightarrow \begin{bmatrix} \exists x_1 \in A : y = f(x_1) \\ \exists x_2 \in B : y = f(x_2) \end{bmatrix}$$
$$\Rightarrow \exists x \in A \cup B \text{ so that } y = f(x) \text{ or } y \in f(A \cup B)$$

- The lecturer asks other students to use the signs for the image of a set, the union of two sets to check the correctness of the inclusion (2).

 $y \in f(A \cup B) \Rightarrow \exists x \in A \cup B : y = f(x)$ 

According to the definition of the image of a set, we have:

$$\Rightarrow \begin{bmatrix} \exists x \in A : y = f(A) \\ \exists x \in B : y = f(B) \end{bmatrix}$$
$$\Rightarrow y \in f(A) \cup f(B)$$

From the proofs of (1) and (2), students draw the theorem.

- The lecturer asks students to write down the hypothesis and conclusions of the theorem and then asks students to express the formula in their own language:  $f(A \cup B) = f(A) \cup f(B)$ 

"The image of set A united with set B is equal to the image of set A united with the image of set B through the mapping f".

- The lecturer asks students to present in natural language on a common method to prove that the two sets are equal.

While presenting by writing on the board, the lecturer asks students to write on the board in their own language for other students to understand and at the same time convert the ideas from spoken to written ML.

*Step 3:* Use ML to apply theorems and train students to practice in-depth study on a mathematical issue.

Situation 1: Ask a student to present the solution to the following problem:

Problem: Given a mapping  $f: R \to R$  determined by the formula  $f(x) = x^2$ .

 $A = \{1\}, B = \{-1\}, C = \{1; 2\}, D = \{4; 5\}$ 

a) Identify the sets  $f(A \cup B)$ ,  $f(A) \cup f(B)$ , f(X | A), f(X) | f(A).

b) Identify the sets  $f(C \cap D)$ ,  $f(C) \cap f(D)$ .

The lecturer asks other students to predict the results of question b and use the Ven chart to illustrate:

 $f(C \cap D) \neq f(C) \cap f(D), f(C \cap D) \subset f(C) \cap f(D).$ 

*Situation 2:* The lecturer asks students to apply the above theorem and use the concepts of intersection of many sets, the associative property of union to solve the problem:

Problem: Let A, B, C be three arbitrary sets, prove that:  $f(A \cup B \cup C) = f(A) \cup f(B) \cup f(C)$ 

When proving that these two sets are equal, students need to use the associative property of the union to represent the union of the three sets as the union of the two sets as follows:  $A \cup B \cup C = (A \cup B) \cup C$ , and then apply the above theorem to the two sets  $A \cup B$ , *C*.

Through the above example, lecturers help SME use symbols, semantics and syntax of the concepts of image, creating images of a set through a mapping, the way to prove that the two sets are equal and at the same time help SME use mathematical reasoning and reasoning in proving theorems. Moreover, through the activities in the above situations, SME can develop the ability to self-study mathematical problems, and understand the method of working in Mathematics, that is, lecturers form the ability for them to practice scientific research. The above activities help SME develop indicators 1.3, 2.2, 3.1, 3.2 of ML competence.

During the teaching process, the lecturer's use of ML directly affects the formation of ML of SME; therefore, lecturers' spoken and written instructions must be brief and concise. Lecturers need to use rational and standard ML in a flexible and creative way for SME to study. Measure 1 can be implemented right during the process of lecturing in class.

# 2.2.2. Measure 2: Train SME to use correct mathematical representations in terms of semantics and syntax when converting from natural language to ML and vice versa in teaching Mathematical Logic

### 2.2.2.1. Purpose of the measure

Encourage SME to use mathematical representations of Mathematical Logic to discover and present some mathematical contents in reality.

Help SME understand the relationship between Mathematical Logic and the reality.

Help SME to use natural and ML in a flexible and creative way in solving math problems, studying maths and doing mathematical research.

The purpose of this measure is to help SME develop indicators 1.1, 1.2, 2.1, 2.2, 2.3 of ML competence.

### 2.2.2.3. Instructions for implementing the measure

First: Lecturers use real-life situations to motivate SME while teaching Mathematical Logic.

Second: Train SME to use correct mathematical representations when practicing the conversion from natural language to ML and vice versa in solving problems in reality.

# 2.2.3. Measure 3: Train SME to use ML when making inferences in learning Math, teaching Math, studying Math through teaching Mathematical Logic

2.2.3.1. Purpose of the measure

Enable SME to correctly use some common deductive rules in learning Math, teaching Math, and doing research in Maths.

Helping SME develop their ability to reason and deduce in presenting ideas and solving problems in their own language.

The purpose of this measure is to help SME develop indicators 1.3, 2.1, 2.2., 3.1, 3.2 of ML competence.

# 2.2.3.2. Scientific basis of the measure

Mathematical logic trains students to use correct meaning of words and sentences when using language and signs in presenting concepts, definitions, statements; proving theorems; presenting solutions to Math problems; and making comments. As a result, students can practice the ability to use language and signs properly and reasonably.

## 2.2.3.3. Instructions on how to implement the measure

First: Train students to think about propositions and propositional functions through teaching Mathematical logic.

Second: Train students to think about conceptual elements in teaching propositional logic.

Third: Train SME to use the rules of logical reasoning correctly in terms of semantics and syntax when teaching Mathematical Logic.

Fourth: Train SME to argue, reason and deduce when presenting or explaining a matter of Mathematics in teaching Mathematical Logic through seminars.

# 2.2.3.4. Considerations when implementing measure 3

These activities need to be done regularly and repeatedly, not only in Mathematical Logic module but also in other Mathematics modules and in professional training at pedagogical universities. This will contribute to professional competence for SME. Lecturers need to be careful when using natural language and ML in communicating with SME, especially during the classroom activities orseminars in self-study sessions. Lecturers should regularly conduct seminars during self-study sessions to help students develop the skills of speaking, writing and presenting about Maths.

# 2.2.4. Measure 4: Exploit and supplement the system of exercises in Mathematical Logic coursebooks to organize learning for SME

# 2.2.4.1. Purpose of the measure

- Lecturers compose sppecial topics to instruct SME to self-study. This will enable students to deepen their knowledge, practice thinking manipulations, logical reasoning and deductive thinking as well as increase the use of ML after class.

- This measure can contribute to developing elements 4 and 5 of ML competence for SME.

# 2.2.4.2. Scientific basis of the measure

The system of exercises in mathematical logic coursebooks has not been exploited and used in the direction of developing ML competence for SME. Therefore, lecturers need to exploit and supplement the system of exercises in the orientation of strengthening the competence of using ML for students.

# 2.2.4.3. Instructions for implementing measures

First: Exploit and supplement the system of exercises which can contribute to developing ML competence for SME after completing each chapter.

Second: Compose special topics to guide students to self-study Mathematics in a way that contributes to the development of ML competence.

Third: Train students to practice the use of information technology in teaching Mathematical Logic in the direction of contributing to the development of ML competence.

# 2.2.4.4. Considerations when implementing measure 4

Specialized topics are designed to help students learn about the relationship between Mathematical Logic and High School Maths, and to train students to flexibly apply ML in learning and doing research in mathematics. Good implementation of this measure is the basis for students to get used to assessing ML competence of their own and of high school students.

# 2.2.5. Measure 5: Train SME how to assess ML competence of their own and of high school students

2.2.5.1. Purpose of the measure: Help SME assess their own use of ML when teaching Mathematical Logic; Help SME discover their limitations in ML when solving math problems, propose solutions to improve their limitations when using ML in learning, teaching or doing research in Mathematics; Help SME discover mistakes of high school students when using ML in Mathematics and take appropriate remedies. This measure aims for SME to develop indicators 5.1, 5.2 of the ML competence.

# 2.2.5.2. Scientific basis of the measure

Most SME do not see the need assessing ML competence of their own and of high school students, and they lack the skills to assess ML competence of their own and of high school students. SME need to understand the above assessment objectives in teaching, from which they will have a sense of assessing ML competence through the process of learning Math in general and Mathematical logic in particular at university.

# 2.2.5.3. Instructions for implementing measure 5

First: By teaching students how to solve problems of Mathematical Logic, lecturers can train SME how to assess ML competence of their own and of high school students.

Second, Lecturers design situations for SME to practice detecting mistakes in the use of ML of their own and of high school students, clearly pointing out the causes and remedies while teaching Mathematical Logic.

Third: Train SME to analyze and process information, and evaluate solutions to math problems of their own and of their classmates.

2.2.5.4. Considerations when implementing measure 5

The assessment of ML competence of SME through teaching Mathematical logic should be conducted regularly during lessons, exercise-correction lessons, seminars and group discussions.

# 2.3. Conclusion for chapter 2

The above measures clearly show the performing methods for lecturers and students during Mathematical Logic lessons. The way of exploiting the illustrative examples is aslo presented clearly. The above measures also confirms the feasibility of the proposed measures in the conditions of teaching Mathematical logic at universities today.

# **Chapter 3. PEDAGOGICAL EXPERIMENT**

### 3.1. Purpose, requirements and contents of the experiment

## 3.1.1. Purpose of the experiment

An pedagogical experiment is conducted to test the validity of the scientific hypothesis proposed in the thesis. Through teaching practice, the feasibility and effectiveness of the measures proposed in chapter 2 are initially evaluated.

# 3.1.2. Requirements of the experiment

The pedagogical experiment must ensure objectivity, suitability for students, and association to the actual situation of teaching in pedagogical universities.

**3.1.3.** *Experiment tasks:* Compile learning materials, reference materials, topicals and conduct experiments on some of the measures mentioned in chapter 2.

## 3.1.4. Experiment contents

**Content 1:** Experiment on teaching some contents of Mathematical logic in university programs in the direction of developing the competence of using ML for SME.

**Content 2:** Experiment on organizing seminars, group discussions on topics related to the contents of the Mathematical logic module in the orientation of developing the competence of using ML for SME.

**Content 3:** Supervise scientific research for some students in the orientation of the research topic.

#### 3.2. Time, subjects, process, and method of evaluating the experimental results

**3.2.1. Time and subjects of the experiment:** Based on the requirements of the thesis, we conduct experiments including the following phases: +) **Phase 1**: From September to October 2017; +) **Phase 2**: From September to October 2018; +) **Phase 3**: In April 2018; +) **Phase 4**: From December 2017 to May 2018

# 3.2.2. Process and methods of implementing the experimental contents

3.2.2.1. The experimental process for phase 1 and phase 2

3.2.2.2. Forms of conducting the experiment

- Measures 1, 2: implemented in the process of formal teaching.

- Measures 3, 4 and 5: integrated into the experiments on measures 1 and 2 and during self-study and semiar sessions.

#### 3.2.3. Considerations about the

# 3.2.4. Methods of assessing the experiment results

3.2.4.1. Assessed contents

a) Students' ability to perceive the lessons in classes proposed in teaching Mathematical Logic.

b) Students' understanding of knowledge, theory, competence of using ML to express and present mathematical reasoning and to solve practical problems.

c) Assess the progress of students through the process of teaching Mathematical Logic and through the end-of-module test.

3.2.4.2. Methods of evaluating the post-experiment results

- a) Testing; b) Observing the classroom;
- c) Interviews d) Mathematical statistics.

# **3.3. Process of the pedagogical experiment**

## 3.3.1. Experiment content 1

# 3.3.3.1. Phase 1: Qualitative evaluation of the experimental results

Through observation and interviews with students after conducting experimental teaching, we found that: The atmosphere of the experimental class was more exciting; the students can express themselves more; they could mobilize and make the most of their natural language and ML to express, explore solutions, and to exchange ideas between groups of students. In the control class, students were quieter and less active and creative in searching for knowledge; they were unwilling to exchange ideas because the lecturer could not really create an environment for students to express themselves.

Lecturers and students gradually became more interested in experimental lessons. Some elements of the ML competence have also been gradually formed (indicators 1.1, 1.2, 1.3, 2.1, 2.2, 3.1.)

# - Quantitative evaluation of the experimental results

Results were obtained from the tests for experimental and control classes in phase 1



# Figure 3.1. Learning results of the experimental and control classes

After conducting the test, it can be seen that the measures presented in chapter 2 really has an impact in contributing to developing the competence of using ML for SME.

# 3.3.3.2. Phase 2: Qualitative evaluation of the experimental results

Through observation and interviews with students after conducting experimental teaching, we found that: The atmosphere of the experimental class was more exciting; the students can express themselves more; they could mobilize and make the most of their natural language and ML to express, explore solutions, and to exchange ideas between groups of students. In the control class, students were quieter and less active and creative in searching for knowledge; they were unwilling to exchange ideas because the lecturer could not really create an environment for students to express themselves. They were unaware of the need to improve ML competence through the situations in the lesson.

# - Quantitative evaluation of the experimental results

The results of the test for the experimental and control classes are the data for us to process and evaluate. Figure 3.2 shows the data on the results of the tests in the second phase.



# Figure 3.2. Learning results of the experimental and control classes

The test results show that the measures presented in chapter 2 really have an impact on contributing to developing the competence of using ML for SME.

**3.3.2.** Experiment content 2: Organize seminars and group discussions on topics related to the contents of the Mathematical logic module to contribute to developing competence of using ML for SME at the end of the first term, academic year 2017 - 2018.

We conducted a seminar with a sample of 35 randomly selected students from the K17 Mathematics Education Class in the elective seminar session. The experiment was conducted in April 2018. The seminar contents were directly compiled and directed by the researcher.

When obtaining the test results before and after the seminar, we have made some initial comments as follows:

# - Qualitative experimental results

+ In the solution to question 2 in test 1, students found it embarrassing to point out errors in using the inference rules that high school students used to prove the problem. SME could not predict mistakes in terms of using ML.

+ The solution to question 2 of test 2 shows that students have already known how to use the rules of representation in propositional logic to convert the model for the problem.



# - Quantitative experimental results

# Figure 3.3. Test scores of students before and after the experiment

Looking at the students' pre-test and post-test results after the seminar, we find that their post-test results are higher than their pre-test results. The number of students with mark 10 before the experiment is 5 out of 35, while the number of marks 10 before the experiment is only 1/35. These results prove the positive impact of the experiment.

# 3.3.3. Experiment content 3: Supervise scientific research for some students in the experimental classroom in the direction of contributing to the development of ML competence for SME

*3.3.3.1.* Student Lai Thi Huyen Trang (K17 Mathematics Education Class) with the research project title "*Drilling the use of mathematical language for high school students while teaching the contents of equations in Algebra 10 textbook*". The research was evaluated excellent (9.7 points) by the scientific council of Hai Phong University. Thus, according to the lecturer's assessment, the student's use of ML has reached level 4.

*3.3.3.2.* Student Nguyen Thi Thu Thuy (K17 Mathematics Education Class) with the research project title "*Drilling the use of mathematical language for high school students through teaching Algebra grdae 11*". The research was evaluated excellent (9.9 points) by the scientific council of Hai Phong University. Thus, the student's use of ML has reached level 4.

# **3.4.** Conclusion on the experiment

In summary, the experimental results show that the scientific hypothesis of the research problem has been tested; the measures proposed in chapter two do contribute to the development of SME's competence of using ML through teaching Mathematical Logic.

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# CONCLUSION AND RECOMMENDATIONS

#### I. Conclusion

The thesis has obtained some results as follows:

- The thesis has reviewed the theoretical issues related to competence, language, ML, ML competence, ML competence of SME; Analyze the content of teaching Mathematical Logic in universities in the direction that contributes to the development of ML competence.

- Identify specific components of the competence of using ML of SME in universities;

- Contribute to clarifying the reality of teaching Mathematical Logic at universities with pedagogy faculties in the direction of contributing to the development of ML competence;

- Propose a system of pedagogical measures to develop the competence of using ML for SME through teaching Mathematical Logic at university. These measures are cohesive, interconnected, complementary and mutually oriented towards the dual goal;

- Conduct pedagogical experiments to illustrate the feasibility and effectiveness of the pedagogical measures proposed in the thesis. Pedagogical experiment results show that the pedagogical measures proposed initially are feasible and achieve good results. The results of the thesis have been "implemented" in the process of training SME in the Mathematical logic module.

Thus, it is possible to confirm that the research purpose has been realized; the research tasks have beeb accomplished and the scientific hypothesis is acceptable.

#### II. Some recommendations after the study

**1. For the Faculty of Mathematics Education:** Provide students with access to professional standards for high school teachers and outcome standards for SME so that they can understand the social requirements of a high school math teacher and they can realize that one of these requirements is developing the competence of using ML.

**2. For the Division of Mathematics Teaching Methods:** Lecturers should pay attention to the selection and combination of methods of teaching mathematics suitable to each student; apply methods of teaching mathematical logic in a way that contributes to developing the competence of using ML for SME; in teaching mathematics, focus on exploiting the characteristics of ML in the modules to contribute to developing the competence of using ML for SME; organize seminars during self-study sessions and teaching practice sessions, which will contribute to developing pedagogical skills for SME before their pedagogical practice periods at high school.